Renormalization group study of a two-valley system with spin splitting

Alexander Punnoose*

Physics Department, City College of the City University of New York, New York, New York 10031, USA

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Renormalization group equations in a two-valley system with valley splitting and intervalley scattering are derived in the presence of spin splitting induced by a parallel magnetic field. The relevant amplitudes in different regimes set by the relative strengths of the spin and valley splittings and the intervalley scattering rate are identified. The range of applicability of the standard formula for the magnetoconductance is discussed.

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I. INTRODUCTION

In two dimensions, an in-plane magnetic field, $B_\parallel$, couples to the spin degrees of freedom leading to spin splitting of the electronic bands. The electron-electron (e-e) interactions between the different spin bands gives rise to a finite magnetoconductance, $\sigma(B_\parallel,T)$, and hence measurement of $\sigma(B_\parallel,T)$ provides a simple and accurate way of determining the effective spin-related interaction strength.[1,2] In a disordered two-dimensional (2D) electron gas, the transport properties at low temperatures, $k_BT \lesssim \hbar/\tau$, are governed by singular diffusive particle-hole propagators,[3] $\mathcal{D}(q,\omega)=1/(D_0q^2+\omega)$. (Here D_0 is the diffusion constant proportional to the elastic scattering time τ.) The spin splitting introduces gaps, proportional to the Zeeman energy $\Delta_z=g\mu_BB_\parallel$, in the propagators with opposite particle-hole spin projections (i.e., the spin-triplet channels with $S_z=\pm1$) thereby cutting off their singularity. While the presence of these gaps lead to negative magnetconductance,[4,5] $\Delta\sigma(B_\parallel,T)=\sigma(B_\parallel,T)-\sigma(0,T)<0$ in the weak-field limit, $\Delta_z \lesssim k_BT$, in the high-field limit the spin bands are well split and the transport is governed entirely by the $S_z=0$ channels, which are insensitive to spin splitting.[6–8] In multivalley systems, such as high mobility silicon inversion layers [Si-metal-oxide-semiconductor field-effect transistors (Si-MOSFETs)], where the analysis carried out in this paper is most relevant, additional gaps proportional to the valley splitting Δ_v and intervalley scattering rates $\Delta_\perp=\hbar/\tau_\perp$ when present are introduced in the propagators.[9,10]

It is well known that the singular nature of $\mathcal{D}(q,\omega)$ leads to a strong enhancement of the e-e scattering amplitudes at low energies.[11] In two dimensions, renormalization group (RG) theory applied to a weakly disordered system has been extremely successful at capturing this scale dependence to all orders in the e-e scattering amplitudes.[8,12] Strong e-e scattering and energy renormalization effects, where the latter takes into account the renormalization of the Stoner enhancement factor, were incorporated into $\Delta\sigma(B_\parallel,T)$ in Refs. 13 and 14. They are generalized here to include the effects of Δ_v and $\Delta_\perp$. A detailed understanding of the interplay of spin and valley gaps on the form of $\sigma(B_\parallel,T)$ in different T intervals can be used to provide robust estimates for $\Delta_v,\Delta_\perp$ and the various e-e scattering amplitudes in the spin-valley scattering channels by fitting to experiments.

The notations for the various diffusion and e-e scattering amplitudes in the spin-valley basis are described below in this section. The situation in the absence of spin splitting (or $\Delta_z \ll k_BT$) has been discussed in detail in Ref. 15. For completeness and to outline the general theoretical methods employed, the RG equations in the temperature regions $\Delta_\perp \lesssim k_BT \lesssim \Delta_v$ and $k_BT \lesssim \Delta_\perp \lesssim \Delta_v$ [labeled as (A) and (B) regions in Fig. 1] are reproduced in Secs. II A and II B, respectively. The classification of the relevant degenerate e-e scattering amplitudes in these regions are shown schematically in Fig. 1. It is assumed throughout that $\Delta_\perp < \Delta_v$, which is found to be the case in high mobility Si-MOSFETs.[16,17] (Typical values for Δ_v are found to be less than 1 K for electron densities greater than 10^{11} cm^{-2}.) The cases involving $\Delta_z \gtrsim k_BT$ are derived in this paper in Secs. II C and II D in the intermediate field regime $\Delta_\perp \lesssim \Delta_z \lesssim \Delta_v$ and the low-field regime $\Delta_z \lesssim \Delta_\perp \lesssim \Delta_v$, respectively. The corresponding regions are labeled as (C) and (D) in Figs. 2 and 3. The strong splitting limit $\Delta_z > \Delta_v$, has been studied in Ref. 18. This paper, together with Ref. 15 and 18, provide the complete RG description of a weakly disordered two-valley system in a parallel magnetic field in the presence of valley splitting and intervalley scattering. It should be noted that the contributions arising from the particle-particle channels, that is, the Cooperon channels, have been suppressed in these calculations, as it is seen experimentally in low-density Si-MOSFETs that the phase breaking rate saturates at low temperatures.[19] It is, nevertheless, simple to include the weak-localization corrections into the final RG equations, as it is known[3] that both spin and valley splittings do not affect the weak-localization contribution to the RG equations to

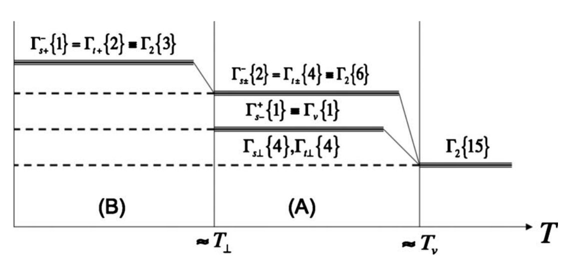

FIG. 1. Schematics showing the classification of the e-e scattering amplitudes as a function of temperature T in the presence of valley splitting T_v and intervalley scattering $T_\perp$. The relevant amplitudes are marked by solid lines with the degenerate amplitudes grouped together. The dashed lines mark the irrelevant amplitudes. For completeness, the RG equations in each of the temperature regimes $T_\perp \lesssim T \lesssim T_v$ and $T \lesssim T_\perp \lesssim T_v$, labeled (A) and (B) in the figure, are reproduced below in Secs. II A and II B, respectively, from Ref. 15.

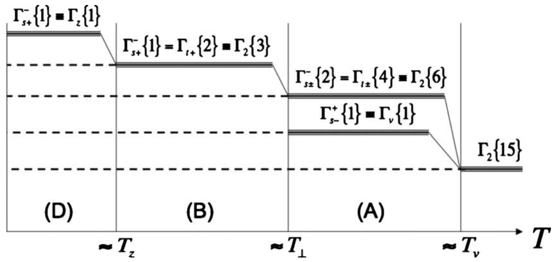

FIG. 2. Schematics showing the classification of the relevant *e-e* scattering amplitudes for weak spin splitting, $T_z \lesssim T_\perp, T_v$, as function of temperature. The RG equations corresponding to region (D), i.e., $T \lesssim T_z$, are derived in Sec. II D.

one-loop order when $k_B T > \Delta_\perp$. In the opposite limit $k_B T \lesssim \Delta_\perp$ the number of Cooperon valley modes reduces to a single mode due to strong intervalley scattering, the weak-localization contribution is therefore halved in this limit.

Electrons in a two-valley system acquire additional valley indices $|\tau\rangle = \pm$ depending on their valley occupancy. This results in $4 \times 4 = 16$ electron-hole states, which may be conveniently combined into spin/valley-"singlet" and spin/valley-"triplet" pairs. The various diffusion propagators and the *e-e* scattering amplitudes in the (spin-singlet/triplets) $\otimes$ (valley-singlet/triplets) basis are described below.

Diffusion modes. For $\Delta_z = 0$, it was sufficient to label the modes in terms of the valley states $\mathcal{D}_\alpha(q,\omega)$, where $\alpha = \pm$ and $\perp$. (See Ref. 15 for further details.) $\alpha = +$ refers to the valley-singlet channel which is gapless, and $\alpha = -$ and $\perp$ refer to the gapped valley-triplet channels with gaps proportional to $\Delta_\perp$ and $\Delta_v + \Delta_\perp$, respectively. Since $\mathcal{D}_-$ involves scattering only within the same valley, it is insensitive to the splitting Δ_v. It, however, develops a gap $\Delta_\perp$ when intervalley scattering mixes the two valleys. All modes are effectively gapless at high temperatures, $k_B T \gtrsim \Delta_v, \Delta_\perp$. [Temperature units T_v, $T_\perp$, and T_z will be used interchangeably in the following to represent the scales Δ_v, $\Delta_\perp$, and Δ_z, respectively.]

For $\Delta_z \neq 0$, the spin-triplet channels $S_z = \pm 1$ develop spin gaps proportional to Δ_z. Hence, it is convenient to label the propagators as $\mathcal{D}_{t\alpha}$ and $\mathcal{D}^\sigma_{s\alpha}$, where the subscript t corresponds to the spin-triplet channels with $S_z = \pm 1$, and s labels the $S_z = 0$ channels, with the singlet and the triplet $S_z = 0$ channels labeled by $\sigma = \pm$.

e-e scattering amplitudes. In a single valley system, the *e-e* scattering amplitudes are uniquely described by the spin texture of the scattering channel. The amplitudes Γ_s and Γ_t are used to describe the scattering of particle-hole pairs in the spin-singlet and spin-triplet channels, respectively. They are related to the standard static Fermi-liquid amplitudes Γ_1 and Γ_2 as $\Gamma_s = \Gamma_1 - \Gamma_2/2$ and $\Gamma_t = -\Gamma_2/2$. These definitions are easily extended to,[15] $\Gamma^\sigma_{s\alpha} = \Gamma_{2\alpha} - 4\Gamma^\sigma_{1\alpha}$ and $\Gamma_{t\alpha} = \Gamma_{2\alpha}$, where $\alpha = \pm, \perp$, and $\sigma = \pm$. [For notational convenience, the amplitudes $\Gamma^\sigma_{s\alpha}$ are defined with a factor of -4.] Note that the intervalley scattering amplitudes $\Gamma^\sigma_{1\perp}$ are generally negligibly small in a clean system because the Coulomb scattering involving large momentum Q_0 perpendicular to the 2D plane is suppressed when the width of the inversion layer is many times larger than the lattice spacing, hence $\Gamma^\sigma_{s\perp} = \Gamma_{t\perp} = \Gamma_{2\perp}$. Together, the total number of amplitudes equal $\Gamma^\sigma_{s\alpha}\{4\} + \Gamma_{t\alpha}\{8\} + \Gamma^\sigma_{s\perp}\{4\} = \{16\}$, where the number of channels are given in curly brackets.

In the high-temperature limit, $T \gtrsim T_v, T_z$, the amplitudes $\Gamma^\sigma_{1\alpha}$, except for Γ^+_{1+}, are identically zero. The Γ^+_{1+} amplitude, which involves scattering in the spin and valley singlet channels, (spin-singlet) $\otimes$ (valley-singlet), is special in that it combines with the long-ranged part of the Coulomb interaction to produce a universal amplitude.[3] [Details are given below in Sec. II A] Hence, all 15 of the 16 amplitudes are equal and evolve as Γ_2. They are shown grouped together when $T \gtrsim T_v$ in Fig. 1.

When $T_\perp \lesssim T \lesssim T_v$, the $\mathcal{D}_\perp$ propagators are gapped, the corrections to $\Gamma_\perp$ are therefore nonsingular and hence irrelevant. On the other hand, the Γ^+_{1-} amplitude in the (spin-singlet) $\otimes$ (valley-triplet) channel, which vanishes at high temperatures, was shown in Ref. 15 to be generated under the RG transformations when $T \lesssim T_v$. (To emphasize that Γ^+_{s-} arises as an independent scaling variable only when the valley subbands are split, it is designated as $\Gamma_v \equiv \Gamma^+_{s-}$.) This is a generic feature of multiband systems with subband splittings, it was first discussed in Ref. 18 in the opposite case $T_v \lesssim T \lesssim T_z$ in which case the relevant amplitude is Γ^-_{1+} where the spin and valley indices are interchanged.

The splitting of the 15 amplitudes below T_v are shown schematically in Fig. 1. The solid horizontal lines mark the relevant amplitudes and the dashed lines mark the irrelevant ones. The degenerate amplitudes under the RG flow are grouped together with the degeneracy indicated in curly brackets. At the lowest temperature $T \lesssim T_\perp$, when the two valleys are strongly mixed, only the valley-singlet propagator $\mathcal{D}_+$ remains gapless. Hence, only the amplitudes in the valley-singlet channel Γ_+ survives.

Clearly, the number of relevant *e-e* scattering amplitudes in a multiband system at a given scale is sensitive to the splitting and the interband scattering rates within the bands. Figure 2 shows schematically the effect of a weak magnetic field $T_z \lesssim T_\perp, T_v$ on the amplitudes. The spin gap suppresses the singular corrections in the spin-triplet channels, hence only Γ^-_{s+} (and Γ^+_{s+}) develops singular diffusion corrections. The amplitude is designated as $\Gamma_z \equiv \Gamma^-_{s+}$ to emphasize that $T \lesssim T_z$.

Finally, Fig. 3 shows schematically the relevant amplitudes for intermediate values of the spin splitting $T_\perp \lesssim T_z \lesssim T_v$. As in Fig. 2, the spin-triplet channels, Γ_t, are irrelevant below T_z due to the gap in the $\mathcal{D}_t$ propagators. As a result, the number of relevant amplitudes reduces from seven for $T \gtrsim T_z$ to three for $T \lesssim T_z$.

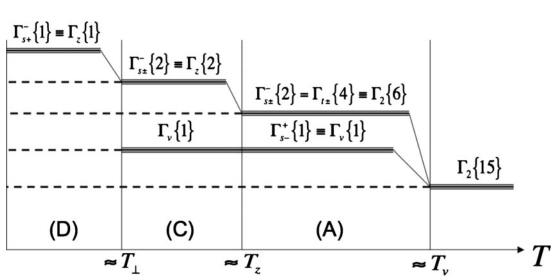

FIG. 3. Schematics showing the classification of the relevant *e-e* scattering amplitudes for intermediate values of the spin splitting, $T_\perp \lesssim T_z \lesssim T_v$, as function of temperature. The RG equations corresponding to region (C), i.e., $T_\perp \lesssim T \lesssim T_z$ are derived in Sec. II C.

II. SCALING EQUATIONS

The RG equations in each of the temperature intervals shown in Figs. 2 and 3 are derived below. The relevant equations when spin splitting can be ignored, $T \gtrsim T_z$, have been derived in detail in Ref. 15. The logarithmic corrections are presented here in Secs. II A and II B after including the spin degrees of freedom explicitly.

A. $T_\perp, T_z \lesssim T \lesssim T_v$

Since the $\mathcal{D}_\perp$ modes are gapped for $T \lesssim T_v$, their contributions are nonsingular and hence dropped. All other modes are effectively gapless when $T \gtrsim T_z, T_\perp$. The gapless propagators are set equal to $\mathcal{D}^\sigma_{s\pm} = \mathcal{D}_{t\pm} \equiv \mathcal{D}(q,\omega) = 1/(Dq^2 + z\omega)$, where D is the renormalized diffusion constant and z parametrizes the relative scaling of the frequency with respect to the length scale.[11,20] Both D and z acquire diffusion corrections in an interacting system. ($z=1$ for a noninteracting system.[3])

The nature of the gapless diffusion modes induce the following relations on the amplitudes: $\Gamma_{t+} = \Gamma_{t-} \equiv \Gamma_2$, $\delta\Gamma^+_{1+} = \delta\Gamma^+_{1-}$, and $\delta\Gamma^-_{1\pm} = 0$. Since $\delta\Gamma^-_{1\pm} = 0$, the amplitudes $\Gamma^-_{s\pm} = \Gamma_2$ are degenerate. The diffusion corrections in terms of these variables take the form[15]

$$\frac{\delta D}{D} = -\frac{4}{\nu} \int\int (\Gamma^+_{1-} + \Gamma^+_{1+} - 2\Gamma_2)\mathcal{D}^3(q,\omega) Dq^2, \tag{1a}$$

$$\delta z = -\frac{1}{\pi\nu}\int \frac{d^2q}{(2\pi)^2}(\Gamma^+_{1-} + \Gamma^+_{1+} - 2\Gamma_2)\mathcal{D}(q,0), \tag{1b}$$

$$\delta\Gamma_2 = \frac{1}{\pi\nu}\int \frac{d^2q}{(2\pi)^2}(\Gamma^+_{1-} + \Gamma^+_{1+})\mathcal{D}(q,0) + 8\Psi(\Gamma_2), \tag{1c}$$

$$\delta\Gamma^+_{1\pm} = \frac{1}{2\pi\nu}\int \frac{d^2q}{(2\pi)^2}\Gamma_2 \mathcal{D}(q,0) + 2\Psi(\Gamma_2). \tag{1d}$$

The single integral is defined as $\int = d^2q/(2\pi)^2$ and the double integral as $\int\int = \int d^2q/(2\pi)^2 \int d\omega/(2\pi)$. The density of states per spin and valley $\nu = m/2\pi$. The contributions of the "ring" diagrams[11] equals $\Psi(\Gamma_2)$, where (see Fig. 5 in Ref. 15)

$$\Psi(\Gamma_2) = +\frac{1}{\nu}\int\int \Gamma_2[\Gamma_2\mathcal{D}^2] - \frac{1}{2}[\Gamma_2^2\mathcal{D}^2] - \frac{1}{\nu}\int\int \omega\Gamma_2[\Gamma_2^2\mathcal{D}^3] - \omega\Gamma_2^2[\Gamma_2\mathcal{D}^3] - \frac{1}{2\nu}\int\int \omega^2\Gamma_2^2[\Gamma_2^2\mathcal{D}^4]. \tag{2}$$

As noted already, the relevance of the Γ^+_{1-} amplitude in the temperature range $T_\perp \lesssim T \lesssim T_v$ is specific to problems with split bands in a multivalley system. Although the corrections $\delta\Gamma^+_{1+} = \delta\Gamma^+_{1-}$ for $T \lesssim T_v$, their initial values are different. The amplitude $\Gamma^+_{1-} = 0$ when $T \gtrsim T_v$, while the singlet amplitude Γ^+_{1+} is special as it combines with the static limit of the Coulomb interaction, denoted here as Γ^+_{0+} (it is conventionally denoted simply as Γ_0 in a single valley system with degenerate spin bands[3]). The Γ^+_{1+} amplitude appearing in Eqs. (1a)–(1c) are to be replaced by its long-ranged value,

$$\Gamma^+_{1+} \to \Gamma^{LR}_1 = \Gamma^+_{0+} + \Gamma^+_{1+}. \tag{3}$$

When combined with the Γ_2 amplitude, the long-ranged singlet amplitude is given as: $\Gamma^{LR}_s = \Gamma_2 - 4\Gamma^{LR}_1$. (To be consistent with the notations in this paper, Γ^{LR}_s is defined with an extra factor of -4.) It is easily verified by combining Eqs. (1b)–(1d) that the singlet combination $\delta(z + \Gamma^{LR}_s) = 0$ is satisfied at all length scales, provided the corrections to the static amplitude $\delta\Gamma^+_{0+} = 0$. This is a well-established result with great importance for the general structure of the theory.[11,12]

Having obtained the leading logarithmic corrections, the scaling equations are derived to first order in the dimensionless resistance $\rho = 1/4(2\pi^2\nu D)$ and to all orders in the e-e interaction amplitudes by performing the ladder summations described in Fig. 6 in Ref. 15. It amounts to replacing the static amplitudes Γ_i by the dynamical amplitudes $U_i(q,\omega)$,

$$U_i(q,\omega) = \Gamma_i \frac{\mathcal{D}_i(q,\omega)}{\mathcal{D}(q,\omega)}, \tag{4}$$

where, the propagators $\mathcal{D}_i$ are defined as

$$\mathcal{D}_i(q,\omega) = \frac{1}{Dq^2 + (z + \Gamma_i)\omega}. \tag{5}$$

The amplitudes Γ_i represents Γ_2, Γ^+_{s-}, and Γ^{LR}_s. Note that since the leading logarithmic corrections involve only one momentum integration, it generates only one factor of $1/D$. The corrections are therefore limited to the first order in resistance ρ (disorder). The limitation on the number of momentum integrations constraints the number of e-e vertices in the skeleton diagrams. The ladder sums extend the skeleton diagrams to all orders in Γ_i without changing the number of momentum integrations. Note, however, that only those interaction vertices involving frequency integrations can be extended to include dynamical effects. These amplitudes are enclosed in square brackets in Eq. (2). Substituting the Γ_2 amplitudes in the square brackets with U_2 and performing the q,ω integrals leads to the very simple expression,[8,11]

$$\Psi(\Gamma_2) = \left(\frac{\Gamma_2^2}{z}\right) \times \frac{\rho}{2}\log\left(\frac{1}{T\tau}\right). \tag{6}$$

The remaining single integrals $\int d^2q\mathcal{D}(q,0)$ involving only momentum integrations are easily evaluated to give

$$\frac{1}{\pi\nu}\int \frac{d^2q}{(2\pi)^2}\mathcal{D}(q,0) = 2\rho\log\left(\frac{1}{T\tau}\right). \tag{7}$$

The integrals in δD containing ω integrations remain to be evaluated. Before the integrals can be done, the Γ^+_{1+} amplitude is replaced with Γ^{LR}_1 following Eq. (3), after which the amplitudes Γ^{LR}_1, Γ^+_{1-}, and Γ_2 are rearranged to form Γ^{LR}_s and Γ^+_{s-} and Γ_2 and then extended to U^{LR}_s, U^+_{s-}, and U_2, respectively.

When the equation for ρ is expressed in terms of the scaling variables, $\gamma_2 = \Gamma_2/z$ and $\gamma_v = \Gamma^+_{s-}/z$, the equations for ρ, γ_2, and γ_v form a closed set of equations independent of z.

The final RG equations in the range $T_\perp, T_z \lesssim T \lesssim T_v$ are given below with the scale ξ defined to logarithmic accuracy as $\xi = \log(1/T\tau)$,

$$\frac{d\rho}{d\xi} = \rho^2[1 - \Phi(\gamma_v) - 6\Phi(\gamma_2)], \tag{8a}$$

$$\frac{d\gamma_2}{d\xi} = \frac{\rho}{2}[(1+\gamma_2)^2 + (1+\gamma_2)(\gamma_2 - \gamma_v)], \tag{8b}$$

$$\frac{d\gamma_v}{d\xi} = \frac{\rho}{2}(1+\gamma_v)(1-\gamma_v - 6\gamma_2), \tag{8c}$$

$$\frac{d\ln z}{d\xi} = -\frac{\rho}{2}(1-\gamma_v - 6\gamma_2). \tag{8d}$$

The function $\Phi(\gamma)$ is defined as

$$\Phi(\gamma) = \left(1 + \frac{1}{\gamma}\right)\log(1+\gamma) - 1. \tag{9}$$

As described in Fig. 3, the 15 degenerate amplitudes for $T \gtrsim T_v$ split into six Γ_2 and one Γ_v amplitude when $T \lesssim T_v$. [This splitting of the amplitudes is generic to multiband systems with subband splittings. The same equations are obtained when instead of the valley bands, the spin bands are split,[18] i.e., $T_v \lesssim T \lesssim T_z$.] Note that γ_v coincides with γ_2 when $T \approx T_v$.

B. $T_z \lesssim T \lesssim T_\perp, T_v$

The relevant amplitudes in the presence of strong valley mixing $(T \lesssim T_\perp)$ correspond to scattering in the valley-singlet channels, Γ_{s+}^- and Γ_{t+}. Since $\delta\Gamma_{1+}^- = 0$ vanishes in the absence of spin splitting $(T \gtrsim T_z)$, it follows that the amplitudes $\Gamma_{s+}^- = \Gamma_{t+} = \Gamma_2$ are all equal and satisfy the equations[15]

$$\frac{\delta D}{D} = -\frac{4}{\nu}\int\int (\Gamma_{1+}^+ - \Gamma_2)\mathcal{D}^3(q,\omega)Dq^2, \tag{10a}$$

$$\delta z = -\frac{1}{\pi\nu}\int \frac{d^2q}{(2\pi)^2}(\Gamma_{1+}^+ - \Gamma_2)\mathcal{D}(q,0), \tag{10b}$$

$$\delta\Gamma_2 = \frac{1}{\pi\nu}\int \frac{d^2q}{(2\pi)^2}\Gamma_{1+}^+\mathcal{D}(q,0) + 4\Psi(\Gamma_2), \tag{10c}$$

$$\delta\Gamma_{1+}^+ = \frac{1}{4\pi\nu}\int \frac{d^2q}{(2\pi)^2}\Gamma_2\mathcal{D}(q,0) + \Psi(\Gamma_2). \tag{10d}$$

The coefficient of Γ_2 and the ring diagrams $\Psi(\Gamma_2)$ in Eqs. (10a)–(10c) are suppressed by a factor 2 when compared with Eqs. (1a)–(1c) since they no longer contain a valley sum. The corrections to $\Gamma_{1\alpha}^\sigma$ in Eq. (10d) already do not carry a valley sum, only half the amplitude involving the same valley, however, acquires corrections when the valley bands are mixed, which accounts for the overall factor of half when compared with Eq. (1d). Note that the condition $\delta(z + \Gamma_s^{LR}) = 0$ is satisfied. Following the procedure described in Sec. II A, the RG equations read

$$\frac{d\rho}{d\xi} = \rho^2[1 - 3\Phi(\gamma_2)], \tag{11a}$$

$$\frac{d\gamma_2}{d\xi} = \frac{\rho}{2}(1+\gamma_2)^2, \tag{11b}$$

$$\frac{d\ln z}{d\xi} = -\frac{\rho}{2}(1 - 3\gamma_2). \tag{11c}$$

The function $\Phi(\gamma)$ is defined in Eq. (9). As described in Fig. 2, only three of the 15 degenerate amplitudes survive when $T \lesssim T_\perp$ when spin splitting can be neglected $T \gtrsim T_z$. The high field cases are discussed below, i.e., $T \lesssim T_z$.

C. $T_\perp \lesssim T \lesssim T_z \lesssim T_v$

It should be noted that the results for $T \lesssim T_z \lesssim T_v$ is equivalent to the situation if the gap scales were reversed, i.e., $T \lesssim T_v \lesssim T_z$, provided of course the spin and valley indices are interchangeable, which is the case when $T \gtrsim T_\perp$. The RG equations for $T_v \lesssim T_z$ are derived in Ref. 18. The opposite situation $T_z \lesssim T_v$ is derived here.

When $T \lesssim T_z$, the $\mathcal{D}_{t\pm}$ propagators are gapped, and hence the corrections in the $S_z = \pm 1$ channel are nonsingular. The corresponding amplitudes $\Gamma_{t\pm}$ are therefore irrelevant at these temperatures, which reduces the number of relevant interaction amplitudes by 4. Furthermore, the amplitude Γ_{1+}^- acquires diffusion corrections[18] when $T \lesssim T_z$ in the same way that Γ_{1-}^+ does when $T \lesssim T_v$. Since $T \lesssim T_z$ and T_v, the amplitude Γ_{1-}^- also acquires logarithmic corrections. As a result, both $\Gamma_{s\pm}^-$ are different from Γ_2 when $T \lesssim T_z, T_v$. After including the contributions from $\Gamma_{1\alpha}^\sigma$, the diffusion corrections for $T \lesssim T_z$ take the form

$$\frac{\delta D}{D} = -\frac{4}{\nu}\int\int \left(\sum_{\alpha,\sigma=\pm}\Gamma_{1\alpha}^\sigma - \Gamma_2\right)\mathcal{D}^3(q,\omega)Dq^2, \tag{12a}$$

$$\delta z = -\frac{1}{\pi\nu}\int \left(\sum_{\alpha,\sigma=\pm}\Gamma_{1\alpha}^\sigma - \Gamma_2\right)\mathcal{D}(q,0), \tag{12b}$$

$$\delta\Gamma_2 = \frac{1}{\pi\nu}\int \sum_{\alpha,\sigma=\pm}\Gamma_{1\alpha}^\sigma\mathcal{D}(q,0) + 4\Psi(\Gamma_2), \tag{12c}$$

$$\delta\Gamma_{1\alpha}^\sigma = \frac{1}{4\pi\nu}\int \Gamma_2\mathcal{D}(q,0) + \Psi(\Gamma_2). \tag{12d}$$

The coefficient of Γ_2 and the ring diagrams $\Psi(\Gamma_2)$ in Eqs. (12a)–(12c) are suppressed by a factor 2 when compared with Eqs. (1a)–(1c) since they no longer contain a spin sum. The corrections to $\Gamma_{1\alpha}^\sigma$ in Eq. (12d) already do not carry a spin sum. Only half the amplitude involving the same spin, however, acquires corrections when the spin bands are split, which accounts for the overall factor of half when compared with Eq. (1d). Since $\delta\Gamma_{1+}^- = \delta\Gamma_{1-}^-$, the amplitudes, after combining with Γ_2, can be grouped together as $\Gamma_{s\alpha}^- \equiv \Gamma_z$. Extending the singlet amplitude Γ_{1+}^+ to include the static long-ranged part of the Coulomb interactions Γ_1^{LR} as discussed in

Eq. (3) and using the identity $\Gamma_2-\Sigma_{\alpha,\sigma}\Gamma^{\sigma}_{1\alpha}=\Sigma_{\alpha,\sigma}\Gamma^{\sigma}_{s\alpha}/4$, the amplitude Γ_2 can be eliminated from Eqs. (12a)–(12d) in favor of the amplitudes Γ^{LR}_s, Γ_z, and Γ_v as

$$\frac{\delta D}{D}=\frac{1}{\nu}\int\int(\Gamma^{LR}_s+\Gamma_v+2\Gamma_z)\mathcal{D}^3(q,\omega)Dq^2, \tag{13a}$$

$$\delta z=\frac{1}{4\pi\nu}\int(\Gamma^{LR}_s+\Gamma_v+2\Gamma_z)\mathcal{D}(q,0), \tag{13b}$$

$$\delta\Gamma_z=\delta\Gamma_v=\delta\Gamma^{LR}_s=-\delta z. \tag{13c}$$

Combining Eqs. (12c) and (12d) to give Eq. (13c) is possible only because the $\Psi(\Gamma_2)$ contribution cancels exactly when the sum over opposite spin projections are suppressed due to spin splitting.[7] Also note in Eq. (13c), that the singlet combination $\delta(z+\Gamma^{LR}_s)=0$ holds explicitly, as needed for the consistency of the RG theory.[8,12]

The RG equations are obtained by evaluating the integrals after extending the static amplitudes by the dynamical amplitudes U_i defined in Eq. (4). The RG equations for $T_\perp\lesssim T\lesssim T_z\lesssim T_v$ are

$$\frac{d\rho}{d\xi}=\rho^2[1-\Phi(\gamma_v)-2\Phi(\gamma_z)], \tag{14a}$$

$$\frac{d\gamma_z}{d\xi}=\frac{\rho}{2}(1+\gamma_z)(1-\gamma_v-2\gamma_z), \tag{14b}$$

$$\frac{d\gamma_v}{d\xi}=\frac{\rho}{2}(1+\gamma_v)(1-\gamma_v-2\gamma_z), \tag{14c}$$

$$\frac{d\ln z}{d\xi}=-\frac{\rho}{2}(1-\gamma_v-2\gamma_z). \tag{14d}$$

As described in Fig. 3, the four $\Gamma_{t\pm}$ amplitudes are suppressed when $T\lesssim T_z$, leaving two Γ_z amplitudes, which evolve away from Γ_2. Note that $\gamma_z\approx\gamma_2$ when $T\approx T_z$, while $\gamma_v\approx\gamma_2$ when $T\approx T_v$. [The RG equations when spin splitting is large, $T\lesssim T_v\lesssim T_z$, take the same form as Eqs. (14a)–(14d) provided the spin and valley indices are interchanged; see Ref. 18 for details.]

D. $T\lesssim T_\perp, T_z, T_v$

The two valleys are strongly mixed when $T\lesssim T_\perp$, leaving only the valley-singlet propagators $\mathcal{D}^{\sigma}_{s+}$ gapless. Hence, only $\Gamma^-_{s+}=\Gamma_z$ and Γ^+_{1+}, survive at low temperatures. The corresponding diffusion corrections read

$$\frac{\delta D}{D}=-\frac{4}{\nu}\int\int\left(\Gamma^-_{1+}+\Gamma^+_{1+}-\frac{1}{2}\Gamma_2\right)\mathcal{D}^3(q,\omega)Dq^2, \tag{15a}$$

$$\delta z=-\frac{1}{\pi\nu}\int\left(\Gamma^-_{1+}+\Gamma^+_{1+}-\frac{1}{2}\Gamma_2\right)\mathcal{D}(q,0), \tag{15b}$$

$$\delta\Gamma_2=\frac{1}{\pi\nu}\int(\Gamma^-_{1+}+\Gamma^+_{1+})\mathcal{D}(q,0)+2\Psi(\Gamma_2), \tag{15c}$$

$$\delta\Gamma^{\sigma}_{1+}=\frac{1}{8\pi\nu}\int\Gamma_2\mathcal{D}(q,0)+\frac{1}{2}\Psi(\Gamma_2). \tag{15d}$$

All terms involving Γ_2 amplitudes are suppressed by a factor of 2 in Eqs. (15a)–(15c) compared to Eqs. (12a)–(12c) due to the suppression of the Γ_{t-} amplitudes, which are irrelevant when $T\lesssim T_\perp$. The equations can be simplified in terms of the amplitudes Γ^{LR}_s and Γ_z as

$$\frac{\delta D}{D}=\frac{1}{\nu}\int\int(\Gamma^{LR}_s+\Gamma_z)\mathcal{D}^3(q,\omega)Dq^2, \tag{16a}$$

$$\delta z=\frac{1}{4\pi\nu}\int(\Gamma^{LR}_s+\Gamma_z)\mathcal{D}(q,0), \tag{16b}$$

$$\delta\Gamma_z=\delta\Gamma^{LR}_s=-\delta z. \tag{16c}$$

Note again that the condition $\delta(z+\Gamma^{LR}_s)=0$ is satisfied. Following the procedure followed in the previous sections, the RG equations for $T\lesssim T_\perp, T_z, T_v$ are

$$\frac{d\rho}{d\xi}=\rho^2[1-\Phi(\gamma_z)], \tag{17a}$$

$$\frac{d\gamma_z}{d\xi}=\frac{\rho}{2}(1+\gamma_z)(1-\gamma_z), \tag{17b}$$

$$\frac{d\ln z}{d\xi}=-\frac{\rho}{2}(1-\gamma_z). \tag{17c}$$

These equations coincide with the results obtained in the case of a single valley with spin splitting studied in Ref. 20. Strong intervalley scattering for $T\lesssim T_\perp$ mixes the two valleys to effectively produce a single valley.

III. CONCLUSIONS

The derivation of the scaling equations in Sec. II were carried out keeping only the gapless valley and spin channels in each temperature interval. The scale dependence of the dimensionless resistance $\rho=(e^2/\pi h)R_\square$, where $R_\square$ is the sheet resistance, is then obtained by integrating the self-consistent set of scaling equations separately in each temperature interval and matching the values of the amplitudes and resistance at the boundaries of each interval. Since the intervals are sensitive to the value of T_z, one obtains in this way $\rho(B_\parallel,T)$ whose inverse gives $\sigma(B_\parallel,T)=1/\rho(B_\parallel,T)$. This method is, however, not accurate as the crossover regions have finite contributions from the gapped channels and are hence nonuniversal.

The case of weak spin splitting $T_z\lesssim T$ can be treated fairly accurately, however. In this case the sensitivity to $B_\parallel$ arises only from the presence of a weak spin gap in the triplet channels below the scale set by T. Hence, subtracting $\sigma(0,T)$ from $\sigma(B_\parallel,T)$ captures only the contributions originating from the suppression of the triplet channels. The explicit form $\Delta\sigma(B_\parallel,T)$ for the single valley case was derived in Refs. 5 and 14 in the limit $T_z\ll T$. When the number, N_t, of Γ_t

amplitudes that develop spin gaps are accounted for, $\Delta\sigma(B_{\parallel},T)$ takes the form

$$\Delta\sigma(B_{\parallel},T) = -0.091\frac{e^2}{2\pi h}N_t\gamma_2(\gamma_2+1)(T_z/T)^2. \quad (18)$$

In Fig. 2, by comparing regions (B) and (D), one observes that both the Γ_{t+} amplitudes develop spin gaps and are suppressed as T_z is varied. It follows that $N_t=2$ when $T_z \lesssim T \lesssim T_{\perp}$. Similar analysis comparing regions (A) and (C) in Fig. 3 gives $N_t=4$ when $T_z \lesssim T \lesssim T_v$. Finally, $N_t=8$ in the high-temperature region $T \gtrsim T_v, T_z$.[1]

To summarize, RG equations in the presence of spin splitting induced by a parallel magnetic field have been obtained in a two-valley system in the valley-split and strong intervalley scattering regimes. The form of $\Delta\sigma(B_{\parallel},T)$ in the weak magnetic field limit are discussed.

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*punnoose@sci.ccny.cuny.edu

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